# Analysis of stresses in a Transversely Isotropic Thin Rotating Disc with rigid inclusion having variable density parameter

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***Abstract:***
*Seth's transition theory is applied to the problems of finitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to transversely isotropic material and isotropic materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca's yield condition. With the introduction of density variation parameter lesser values of angular speed is required to yield at the internal surface for rotating disc made of isotropic /Transversely isotropic materials . The radial stress is maximum at the internal surface for both isotropic and transversely isotropic materials. With the effect of density variation parameter increases the values of stresses and displacement of the rotating disc made of isotropic/transversely isotropic materials. Radial stresses is maximum at the internal surface for both isotropic and transversely isotropic materials.*

***Keywords:*** *Disc, Shaft, Transversely Isotropic, Stresses, Displacement, Yielding.*

## Introduction

This paper is concerned with finitesimal deformation of rotating thin circular disk made of transversely isotropic material. There are many applications of rotating disks in science and engineering. As typical examples, we mention, steam and gas turbines, rotors, compressors, flywheels, computer disc drives and high speed gear engine etc. In the design of modern structures, increasing use is being made of materials which are transversely isotropic. The analysis of stress distribution in the circular disk rotating is important for a better understanding of the behavior and optimum design of structures. Solution for thin isotropic discs can be found in most of the standard elasticity and plasticity standard text books [1-8]. Pooja Mahajan [9] analyzed the problem of eElastic-plastic transition in a transversely isotropic thin rotating disc having variable density with edge loading by using Seth's transation theory. Guven [10] found the elastic - Plastic rotating disk with rigid inclusion under the assumption of Tresca's yield condition, its associated flow rule and linear strain hardening. To obtain the stress distribution, Guven matched the elastic plastic stresses at the same radius $r = z$ of the disc. Perfect elasticity and ideal plasticity are two extreme properties of the material and the use of an ad-hoc rule like yield condition amounts to divide the two extreme properties by a sharp line which is not physically possible. When a material passes from one state to another qualitatively different state, transition takes place. Since this transition is non-linear in character and difficult to investigate, workers have taken certain ad hoc assumptions like yield condition, incompressibility condition and a strain law, which may or may not valid for the problem. Sharma *etl*. [12] solved problems in elastic-plastic transition of transversely isotropic thin rotating disc by using Seth's transition theory. Seth's transition theory [13] does not require these assumptions and thus poses and solves a more general problem, from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of generalized strain measure and asymptotic solution at the critical points or turning points of the differential equations defining the deformed field and has been successfully applied to lager number of the problems [9-15]. Seth has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{\frac{n}{2}}\right], \quad (i = 1, 2, 3) \tag{1}$$

where $n$ is the measure and $\overset{A}{e}_{ii}$ are the Almansi finite strain components. In this research paper, we investigate the problem of finitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft by using Seth's transition theory. The density of the disc is assumed to vary along the radius in the form

$$\rho = \rho_0 \left(r / b\right)^{-m} \tag{1)*$$

where $\rho_0$ is the density at $r =$ b and $m$ is the density parameter. Results have been discussed numerically and depicted graphically.

## MATHEMATICAL MODEL

Let us we consider a thin circular disc of variable density with central bore of radius $a$ and external radius $b$. The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed $\omega$ about an axis perpendicular to its plane and passed through the center as shown in fig. 1. The thickness of disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress $T_{zz}$ is zero.

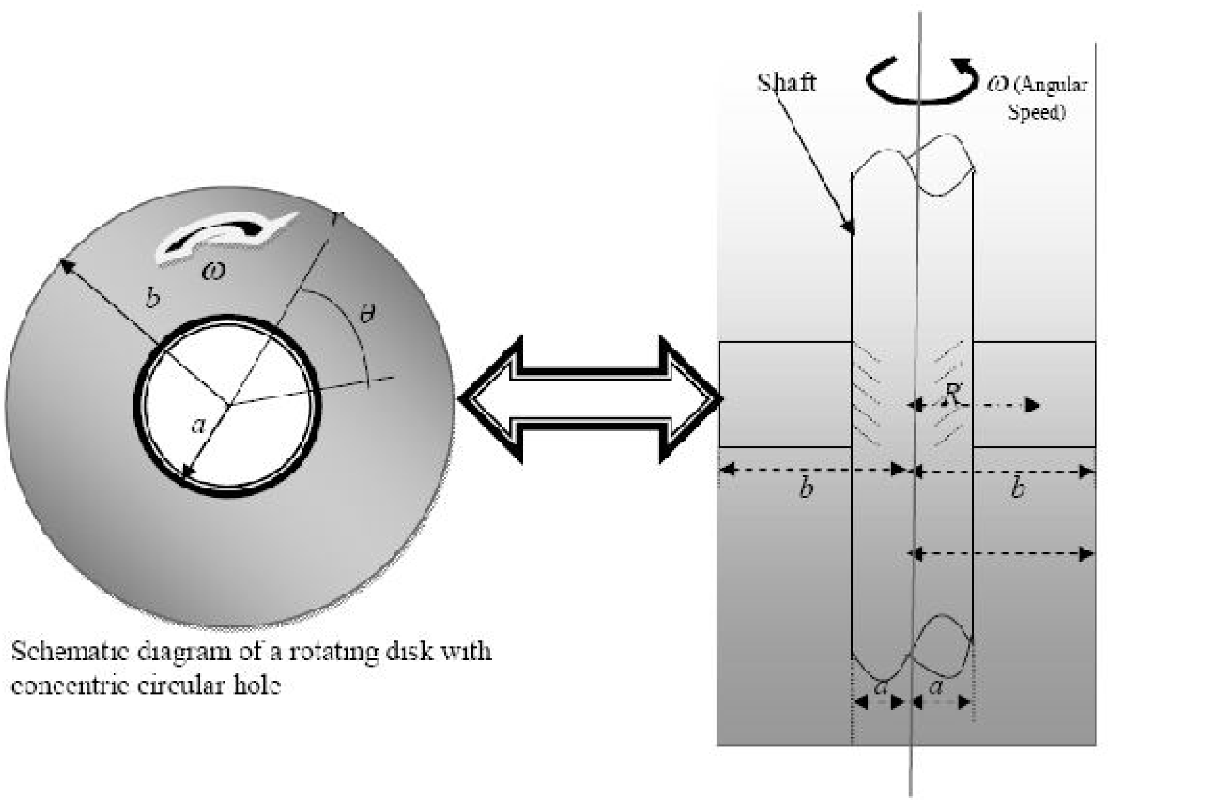


Figure 1. Geometry of Rotating Disc with shaft.

## Boundary conditions

The disk considered in the present study having variable density and mounted on a rigid shaft. The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk is applied mechanical load. Thus, the boundary conditions of the problem are given by:

(i) $r = a$ , $u = 0$

(ii) $r = b$, $T_{rr} = 0$ (2)

where $u$ and $T_{rr}$ denote displacement and stress along the radial direction.

## Governing equations

The components of displacement in cylindrical polar co-ordinates are given by:

$$u = r(1-\beta);\ v = 0;\ w = dz \tag{3}$$

where $\beta$ is position function, depending on $r = \sqrt{x^2 + y^2}$ only, and $d$ is a constant.

The finite strain components are given by [13] as:

$$\overset{A}{e}_{rr} = \frac{1}{2}\left[1-\left(\beta + r\beta'\right)^2\right], \overset{A}{e}_{\theta\theta} = \frac{1}{2}\left[1-\beta^2\right], \overset{A}{e}_{zz} = \frac{1}{2}\left[1-(1-d)^2\right], \overset{A}{e}_{r\theta} = \overset{A}{e}_{\theta z} = \overset{A}{e}_{zr} = 0. \tag{4}$$

where $\beta' = d\beta/dr$ and meaning of superscripts "$A$" is Almansi.

By substituting equation (4) into equation (1), the generalized components of strain are:

$$e_{rr} = \frac{1}{n}\left[1-\left(\beta + r\beta'\right)^n\right], e_{\theta\theta} = \frac{1}{n}\left[1-\beta^n\right], e_{zz} = \frac{1}{n}\left[1-(1-d)^n\right], e_{r\theta} = e_{\theta z} = e_{zr} = 0. \tag{5}$$

The stress-strain relations for transversely isotropic material are given [31]:

$$T_{rr} = C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}e_{zz}, T_{\theta\theta} = (C_{11} - 2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz},$$
$$T_{zz} = C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz} = 0, T_{zr} = T_{\theta z} = T_{r\theta} = 0. \tag{6}$$

Using equation (4) in equation (6), the strain components in terms of stresses are obtained as:

$$e_{rr} \equiv \frac{\partial u}{\partial r} - \frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2 = \frac{1}{2}\left[1-\left(r\beta' + \beta\right)^2\right] = \frac{1}{E}\left[T_{rr} - \left(\frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2}\right)T_{\theta\theta}\right],$$

$$e_{\theta\theta} \equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2}\left[1-\beta^2\right] = \frac{1}{E}\left[T_{\theta\theta} - \left(\frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2}\right)T_{rr}\right],$$

$$e_{zz} \equiv \frac{\partial w}{\partial z} - \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 = \frac{1}{2}\left[1-(1-d)^2\right] = -\frac{1}{E}\left(\frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2}\right)\left[T_{rr} - T_{\theta\theta}\right],$$

$$e_{r\theta} = e_{\theta z} = e_{zr} = 0. \tag{7}$$

where $E = 4C_{66}\left(\frac{C_{11}C_{33} - C_{13}^2 - C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2}\right)$ is *Young's* modulus.

By substituting equations (5) into equations (6), we get:

$$T_{rr}=\frac{A}{n}\left[2-\beta^{n}\left\{1+\left(1+P\right)^{n}\right\}\right]-2\frac{C_{66}}{n}\left[1-\beta^{n}\right],T_{\theta\theta}=\frac{A}{n}\left[2-\beta^{n}\left\{\begin{matrix}1+\\ \left(1+P\right)^{n}\end{matrix}\right\}\right]-2\frac{C_{66}}{n}\left[\begin{matrix}1-\beta^{n}\\ \left(1+P\right)^{n}\end{matrix}\right],$$

$$T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0. \tag{8}$$

where $A=C_{11}-\left(C_{13}^{2}/C_{33}\right)$.

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr}\left(rT_{rr}\right)-T_{\theta\theta}+\rho\omega^{2}r^{2}=0 \tag{9}$$

where $\rho$ is the density of material.

By substituting equations (8) into equation (9), we gets a non-linear differential equation with respect to $\beta$:

$$\beta^{n+1}P(1+P)^{n-1}\frac{dP}{d\beta}=\left[\frac{\rho\omega^{2}r^{2}}{A}+\beta^{n}\left\{\frac{2C_{66}}{nA}\left[1+nP-(1+P)^{n}-P\left\{1+(1+P)^{n}\right\}\right]\right\}\right] \tag{10}$$

where $r\beta'=\beta P$ ($P$ is function of $\beta$ and $\beta$ is function of $r$). The transition points of $\beta$ in equation (10) are $P\to-1$ and $P\to\pm\infty$.

## Solution of the Problem

It has been shown that the asymptotic solution through the principal stress [10, 13 - 34] leads from elastic to plastic state at the transition point $P\to\pm\infty$. If the transition function $R$ is defined as:

$$R\equiv T_{\theta\theta}=\frac{A}{n}\left[2-\beta^{n}\left\{1+\left(1+P\right)^{n}\right\}\right]-2\frac{C_{66}}{n}\left[1-\beta^{n}\left(1+P\right)^{n}\right] \tag{11}$$

Taking the logarithmic differentiation and substitute the value of $dP/d\beta$ from equation (10) in equation (11), one gets:

$$\frac{d}{dr}(\log R)=-\frac{A}{rR}\begin{pmatrix}-\beta^{n}P\left\{1+\left(1+P\right)^{n}\right\}+\frac{2C_{66}}{nA}\beta^{n}-\frac{2C_{66}}{nA}\beta^{n}\left(1+P\right)^{n}+\frac{4C_{66}}{A}P\beta^{n}\\ +\frac{\rho\omega^{2}r^{2}}{A}-\frac{4C_{66}^{2}}{nA^{2}}\beta^{n}\left\{1-\left(1+P\right)^{n}\right\}-\frac{4C_{66}^{2}}{A^{2}}P\beta^{n}-\frac{2C_{66}}{A^{2}}\rho\omega^{2}r^{2}\end{pmatrix} \tag{12}$$

Asymptotic value of equation (12) as $P\to\pm\infty$ and integrating, we get

$$R=Ar^{-C_{2}}\ . \tag{13}$$

where $C_{2}=2C_{66}/A$, $A=C_{11}-\left(C_{13}^{2}/C_{33}\right)$ and $A$ is a constant of integration, which can be determined by the boundary condition.

Using equation (13) in equation (11), we have

$$T_{\theta\theta}=Ar^{-C_{2}} \tag{14}$$

By substituting equations (14) and (1)* into equation (9), one gets:

$$T_{rr} = \frac{B}{r} + A\frac{r^{-C_2}}{1-C_2} - \frac{\rho_0 \omega^2 r^{2-m}}{b^{-m}(3-m)} \tag{15}$$

where $B$ is a constant of integration, which can be determined by the boundary condition. Substituting equation (14) and (15) in second equation of (7) and using equation (1)*, we get:

$$\beta = \sqrt{1 - \frac{2(1-C_2)}{E}\left[\frac{\rho_0 \omega^2 r^{2-m}}{(3-m)} - \frac{K_2}{r}\right]} \tag{16}$$

Substituting equation (16) in equation (3), we get

$$u = r - r\sqrt{1 - \frac{2(1-C_2)}{E}\left[\frac{\rho_0 \omega^2 r^{2-m}}{(3-m)b^{-m}} - \frac{K_2}{r}\right]}, \tag{17}$$

where $1-C_2 = \left(\frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2}\right), E = 2C_{66}(2-C_2)$. is the *Young's* modulus. By applying boundary conditions (2) in equations (15) and (17), we gets: $B = \rho_0 \omega^2 a^{3-m} / (3-m)b^{-m}$ , $A = \frac{\rho_0 \omega^2 \left(b^{3-m} - a^{3-m}\right)(1-C_2)}{(3-m)b^{1-m}b^{-C_2}}$ . Substituting values of constants $A$ and $B$ in equations (14), (15), and (17) respectively, we get the transitional stresses and displacement as:

$$T_{\theta\theta} = \frac{\rho_0 \omega^2 \left(b^{3-m} - a^{3-m}\right)(1-C_2)}{(3-m)b^{1-m}}\left(\frac{r}{b}\right)^{-C_2}, \tag{18}$$

$$T_{rr} = \frac{\rho_0 \omega^2}{(3-m)b^{-m} r}\left[\left(b^{3-m} - a^{3-m}\right)\left(\frac{r}{b}\right)^{1-C_2} + a^{3-m} - r^{3-m}\right], \tag{19}$$

$$u = r - r\sqrt{1 - \frac{2(1-C_2)\rho_0 \omega^2 \left(r^{3-m} - a^{3-m}\right)}{b^{-m}(3-m)Er}}. \tag{20}$$

From equations (18) and (19), we get

$$T_{rr} - T_{\theta\theta} = \frac{\rho_0 \omega^2}{(3-m)b^{-m} r}\left[\left(b^{3-m} - a^{3-m}\right)\left(\frac{r}{b}\right)^{1-C_2} C_2 + a^{3-m} - r^{3-m}\right] \tag{21}$$

***Initial Yielding***: From equation (21), it is seen that $\left|\tau_{rr} - \tau_{\theta\theta}\right|$ is maximum at the internal surface (that is at $r = a$), therefore yielding will take place at the internal surface of the disc and equation (21) gives:

$$\left|T_{rr} - T_{\theta\theta}\right|_{r=a} = \left|\frac{\rho_0 \omega^2 \left(b^{3-m} - a^{3-m}\right)}{(3-m)b^{-m} a}\left(\frac{a}{b}\right)^{1-C_2} C_2\right| \equiv Y(say).$$

where $Y$ is the yielding stress. Angular velocity $\omega_i$ required for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho_0 \omega_i^2 b^2}{Y} = \left| \frac{(3-m) b^{-m} a b^2}{\left(b^{3-m} - a^{3-m}\right) C_2 . (a/b)^{1-C_2}} \right| \tag{22}$$

and $\omega_i = \dfrac{\Omega_i}{b} \sqrt{\dfrac{Y}{\rho_0}}$.

***Fully-plastic state:*** The disc become fully plastic state $(C_2 \to 0)$ at the external surface and equations (21) becomes:

$$\left|T_{rr} - T_{\theta\theta}\right|_{r=b} = \left| \frac{\rho_0 \omega^2 \left(a^{3-m} - b^{3-m}\right)}{(3-m) b^{1-m}} \right| \equiv Y^* (say).$$

where $Y^*$ is the yielding stress. The angular velocity $\omega_f$ for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y^*} = \left| \frac{(3-m) b^{3-m}}{\left(a^{3-m} - b^{3-m}\right)} \right| \tag{23}$$

where $\omega_f = \left(\Omega_f / b\right) \sqrt{Y^* / \rho}$. We introduce the following non-dimensional components as: $R = r/b$, $R_0 = a/b$ $\sigma_r = T_{rr}/Y$, $\sigma_\theta = T_{\theta\theta}/Y$, $H^* = \mathrm{Y}^*/E$, $H = Y/E$ and $U = u/b$. Elastic-plastic transitional stresses, displacement and angular speed from equations (18),(19), (20) and (22) in non-dimensional form become:

$$\left.\begin{aligned} &\sigma_\theta = \frac{\Omega_i^2 \left(1 - R_0^{3-m}\right)(1 - C_2)}{(3-m)} R^{-C_2}, \sigma_r = \frac{\Omega_i^2}{(3-m) R}\left[\left(1 - R_0^{3-m}\right) R^{1-C_2} + R_0^{3-m} - R^{3-m}\right], \\ &U = R - R\sqrt{1 - \frac{2(1-C_2)\Omega_i^2 H \left(R^{3-m} - R_0^{3-m}\right)}{(3-m) R}} \end{aligned}\right] \forall m \neq 3 \tag{24}$$

and $$\Omega_i^2 = \left| \frac{(3-m) R_0}{\left(1 - R_0^{3-m}\right) C_2 . R_0^{1-C_2}} \right|, \quad \forall m \neq 3 \tag{25}$$

For $m = 3$, stresses, displacement and angular speed becomes:

$$\sigma_\theta = -\Omega_i^2 (1 - C_2) R^{-C_2} R_0^{3-m} \ln R_0, \sigma_r = -\frac{\Omega_i^2}{R}\left[R_0^{3-m} . \ln R_0 . R^{1-C_2} - \ln R_0 . R_0^{3-m} + R^{3-m} . \ln R\right],$$

$$\sigma_r = -\frac{\Omega_i^2}{R}\left[R_0^{3-m} . \ln R_0 . R^{1-C_2} - \ln R_0 . R_0^{3-m} + R^{3-m} . \ln R\right], \ \Omega_i^2 = \left| \frac{-R_0^{C_2}}{R_0^{3-m} \ln R_0 . C_2} \right|$$

Stresses, displacement and angular speed for fully- plastic state $\left(C_2 \to 0\right)$ are obtained from equation (24) and (23) become:

$$\sigma_\theta = \frac{\Omega_f^2\left(1-R_0^{3-m}\right)}{(3-m)},\ \sigma_r = \frac{\Omega_f^2}{(3-m)R}\left[\left(1-R_0^{3-m}\right)R + R_0^{3-m} - R^{3-m}\right],$$

$$U = R - R\sqrt{1-\frac{2\Omega_f^2 H^*\left(R^{3-m}-R_0^{3-m}\right)}{(3-m)R}},\ \Omega_f^2 = \left|-\frac{(3-m)}{\left(1-R_0^{3-m}\right)}\right| \quad \forall\, m \neq 3 \tag{26}$$

For $m$ = 3, stresses, displacement and angular speed for fully plastic state become:

$$\sigma_\theta = -\Omega_f^2 R_0^{3-m}.\ln R_0,\ \sigma_r = -\frac{\Omega_f^2}{R}\left[R_0^{3-m}.R.\ln R_0 + \ln R_0.R_0^{3-m} - R^{3-m}.\ln R\right],$$

$$U = R - R\sqrt{1-\frac{2\Omega_f^2 H^*\left(R^{3-m}.\ln R - R_0^{3-m}.\ln R_0\right)}{R}},\ \Omega_f^2 = \left|\frac{-1}{R_0^{3-m}\ln R_0}\right|.$$

**Isotropic Case:** For isotropic materials, the material constants reduce to two only, *i.e.* $C_{11} = C_{22} = C_{33}$, $C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} - 2C_{66})$, and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ .In term of constants $\lambda$ and $\mu$, these can be written as:

$$C_{12} = \lambda,\ C_{11} = \lambda + 2\mu \text{ and } C_{66} = \frac{1}{2}\left(C_{11} - C_{12}\right) \equiv \mu \tag{27}$$

Elastic-plastic transitional stresses, displacement and angular speed are obtained by using equation (27) in equations (18)-(20) and (22) as:

$$\left.\begin{array}{l}\sigma_\theta = \dfrac{\Omega_i^2\left(1-R_0^{3-m}\right)(1-C)}{(3-m)(2-C)} R^{-\frac{1}{2-C}},\sigma_r = \dfrac{\Omega_i^2}{(3-m)R}\left[\left(1-R_0^{3-m}\right)R^{\frac{1-C}{2-C}} + R_0^{3-m} - R^{3-m}\right] \\ U = R - R\sqrt{1-\dfrac{2(1-C)\Omega_i^2 H\left(R^{3-m}-R_0^{3-m}\right)}{(3-m)(2-C)R}}\end{array}\right] \forall\, m \neq 3 \tag{28}$$

$$\text{and}\quad \Omega_i^2 = \left|\frac{(3-m)(2-C)R_0^{\frac{1}{2-C}}}{\left(1-R_0^{3-m}\right)}\right|, \forall\, m \neq 3 \tag{29}$$

where $C = 2\mu/(\lambda + 2\mu), 1 - C_2 = \left(1 - C/2 - C\right)$ .

For $m$ = 3, stresses, displacement and angular speed become:

$$\sigma_\theta = -\frac{\Omega_i^2 R_0^{3-m}.\ln R_0.(1-C)}{(2-C)} R^{-\frac{1}{2-C}}, \sigma_r = -\frac{\Omega_i^2}{R}\left[R_0^{3-m}.\ln R_0.R^{\frac{1-C}{2-C}} - R_0^{3-m}.\ln R_0 + R^{3-m}.\ln R\right]$$

$$U = R - R\sqrt{1 - \frac{2(1-C)\Omega_i^2 H\left(R^{3-m}.\ln R - R_0^{3-m}.\ln R_0\right)}{(2-C)R}}$$

$$\Omega_i^2 = \left|\frac{-(2-C)R_0^{\frac{1}{2-C}}}{R_0^{3-m}.\ln R_0}\right|.$$

**Fully-plastic state (isotropic case):** For fully plastic state ($C \to 0$), equation (28) becomes:

$$\sigma_\theta = \frac{\Omega_f^2\left(1-R_0^{3-m}\right)}{2(3-m)\sqrt{R}}, \ \sigma_r = \frac{\Omega_f^2}{(3-m)R}\left[\left(1-R_0^{3-m}\right)\sqrt{R} + R_0^{3-m} - R^{3-m}\right],$$

$$U_f = R - R\sqrt{1 - H^*\left[\frac{\Omega_f^2}{(3-m)R}\left(R^{3-m} - R_0^{3-m}\right)\right]} \qquad \forall\, m \neq 3 \qquad (30)$$

For $m$ = 3, stresses, displacement and angular speed become:

$$\sigma_\theta = -\frac{\Omega_f^2 R_0^{3-m}.\ln R_0}{2\sqrt{R}}, \sigma_r = -\frac{\Omega_f^2}{R}\left[R_0^{3-m}.\ln R_0.\sqrt{R} - R_0^{3-m}.\ln R_0 + R^{3-m}.\ln R\right],$$

$$U = R - R\sqrt{1 - \frac{\Omega_f^2 H^*\left(R^{3-m}.\ln R - R_0^{3-m}.\ln R_0\right)}{R}}.$$

The disc become fully plastic state $\left(C_2 \to 1/2 \; or \; C \to 0\right)$ at the external surface and equations (21) becomes:

$$\left|T_{rr} - T_{\theta\theta}\right|_{r=b} = \left|\frac{\rho_0 \omega^2\left(b^{3-m} - a^{3-m}\right)}{2(3-m)b^{1-m}}\right| \equiv Y^*(Yielding).$$

where $Y^*$ is the yielding stress. The angular velocity $\omega_f$ for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho_0 \omega_f^2 b^2}{Y^*} = \left|\frac{2(3-m)}{\left(1-R_0^{3-m}\right)}\right|, \ \forall\, m \neq 3 \qquad (31)$$

and $\left|\frac{-2}{R_0^{3-m}.\ln R_0}\right|, \ \forall\, m = 3$

where $R_0 = a/b$ and $\omega_f = \left(\Omega_f / b\right)\sqrt{Y^* / \rho}$.

**Table 1.** Elastic constants $C_{ij}$ (in units of $10^{10}$ N/m$^2$)

| Materials | $C_{44}$ | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{33}$ |
|---|---|---|---|---|---|
| Transversely Isotropic Material ($C_2$ =0.69, Beryl) | 0.883 | 2.746 | 0.980 | 0.674 | 4.69 |
| Isotropic Material ($\sigma = 0.33, C_2 = 0.50$, Brass) | 0.999997 | 3.0 | 1.0 | 1.0 | 3.0 |
| Transversely Isotropic Material ($C_2$ =0.64, Magnesium) | 1.64 | 5.97 | 2.62 | 2.17 | 6.17 |

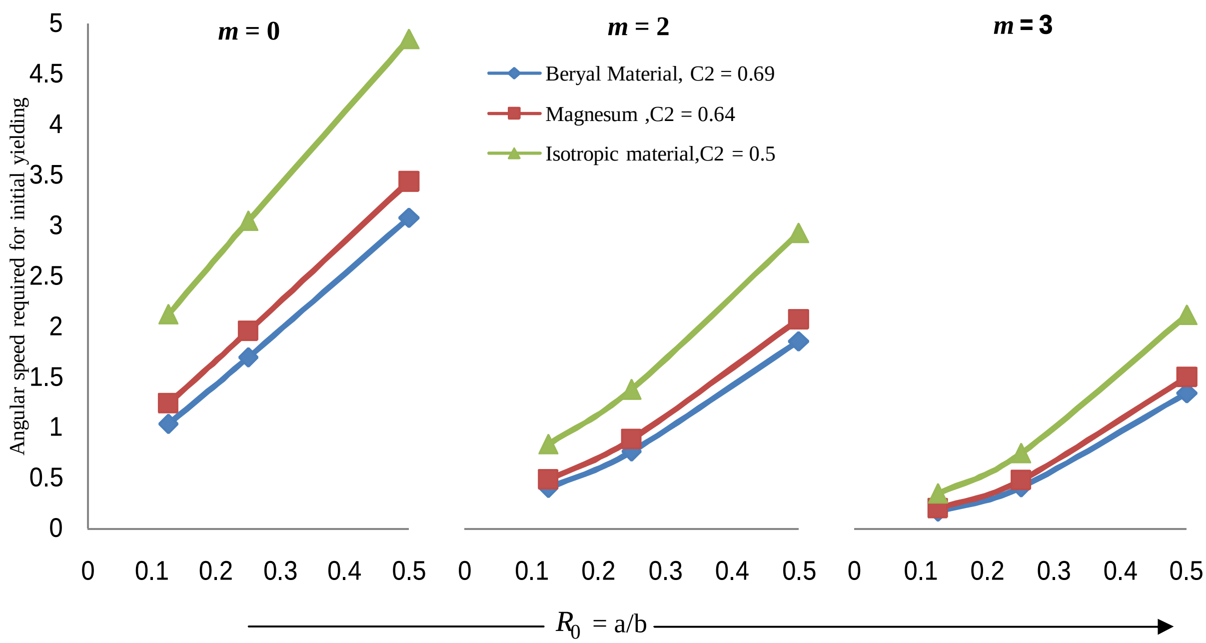


**Figure 2** Angular speed required for initial yielding along the radii ratio $R_0$ = a/b.

**Table 2.** Angular speed required for initial yielding and fully plastic state.

| | Materials | | $m$ | $C_2$ | Angular Speed required for initial yielding $\Omega_i^2$ | Angular Speed required for fully-plastic state $\Omega_f^2$ | Percentage increase in Angular speed $\left(\sqrt{\frac{\Omega_f^2}{\Omega_i^2}}-1\right)\times 100$ |
|---|---|---|---|---|---|---|---|
| 0.5 < R < 1 | Magnesium | Transversely Isotropic Material | 0<br>2<br>3 | 0.64 | 3.437748<br>2.005353<br>1.46556 | 3.428571<br>2<br>1.442695 | -0.1335632 %<br>-0.133557%<br>-0.133557% |
| | Beryl | | 0<br>2<br>3 | 0.69 | 3.080019<br>1.796678<br>1.296029 | 3.428571<br>2<br>1.442695 | 5.5066602%<br>5.5066595%<br>5.506667% |
| | Brass | Isotropic Material | 0<br>2<br>3 | 0.5 | 4.848732<br>2.828427<br>2.044028 | 6.857143<br>4<br>2.88539 | 18.920715%<br>18.920714%<br>18.811602% |

## Numerically Discussion

As a numerical example, elastic constants $C_{ij}$ have been given in table 1 for transversely isotropic materials (Magnesium and Beryl) and isotropic material (Brass, $\sigma = 0.33$ ). Curves have been drawn in figure 2 between angular speed $\Omega_i^2$ required for initial yielding along the radii ratios $R_0$ = a/b. It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. With the introduction of density variation parameter lesser values of angular speed is required to yield at the internal surface for rotating disc made of isotropic /Transversely isotropic materials . It can also be seen from table 2, that rotating disc made of isotropic material ( *i.e.* Brass) required high percentage increase in angular speed to become fully plastic as compared to rotating disc made of transversely isotropic materials (*i.e*. Beryal and Magnesium). In figure 3 and 4, curves have been drawn between stresses distribution and displacement for initial yielding and fully plastic state along the radius ratio $R = r / b$ . Form fig. 3- fig. 4, it has been observed that isotropic material required maximum stresses and displacement as compare to transversely isotropic materials. The radial stress is maximum at the internal surface for both isotropic and transversely isotropic materials. With the effect of density variation parameter increases the values of stresses and displacement of the rotating disc made of isotropic/transversely isotropic materials. Therefore, rotating disc made of transversely isotropic material is on the safer side of the design as compared to disc made isotropic material.

## Conclusion

It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. With the introduction of density variation parameter lesser values of angular speed is required to yield at the internal surface for rotating disc made of isotropic /Transversely isotropic materials . The radial stress is maximum at the internal surface for both isotropic and transversely isotropic materials. With the effect of density variation parameter increases the values of stresses and displacement of the rotating disc made of isotropic/transversely isotropic materials.

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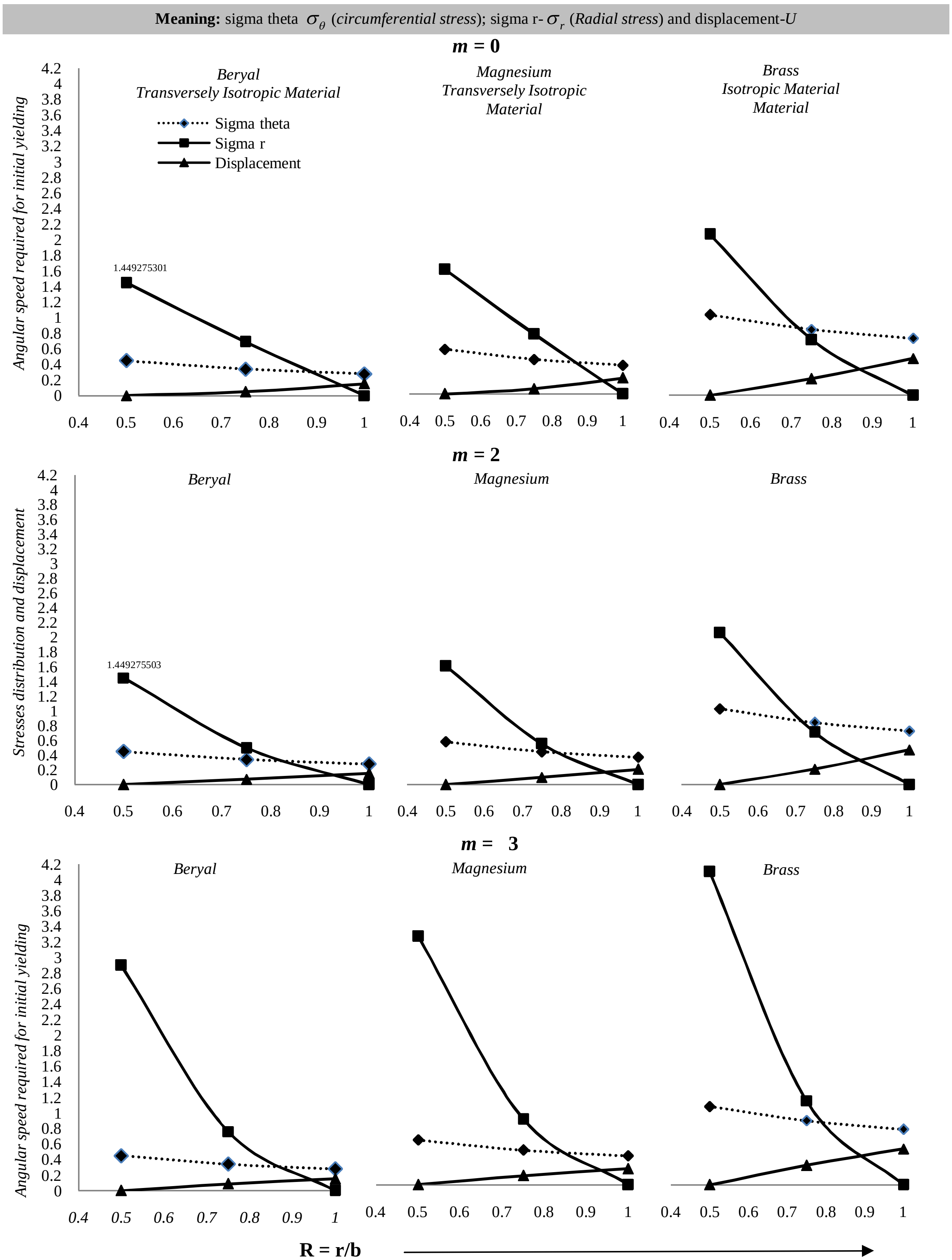


**Figure 3** Stresses distribution and displacement for initial yielding having variable density parameter $m$ = 0, 2, 3 along the radius ratio R = r/b.

**Meaning:** sigma theta $\sigma_\theta$ (*circumferential stress*); sigma r- $\sigma_r$ (*Radial stress*) and displacement-*U*

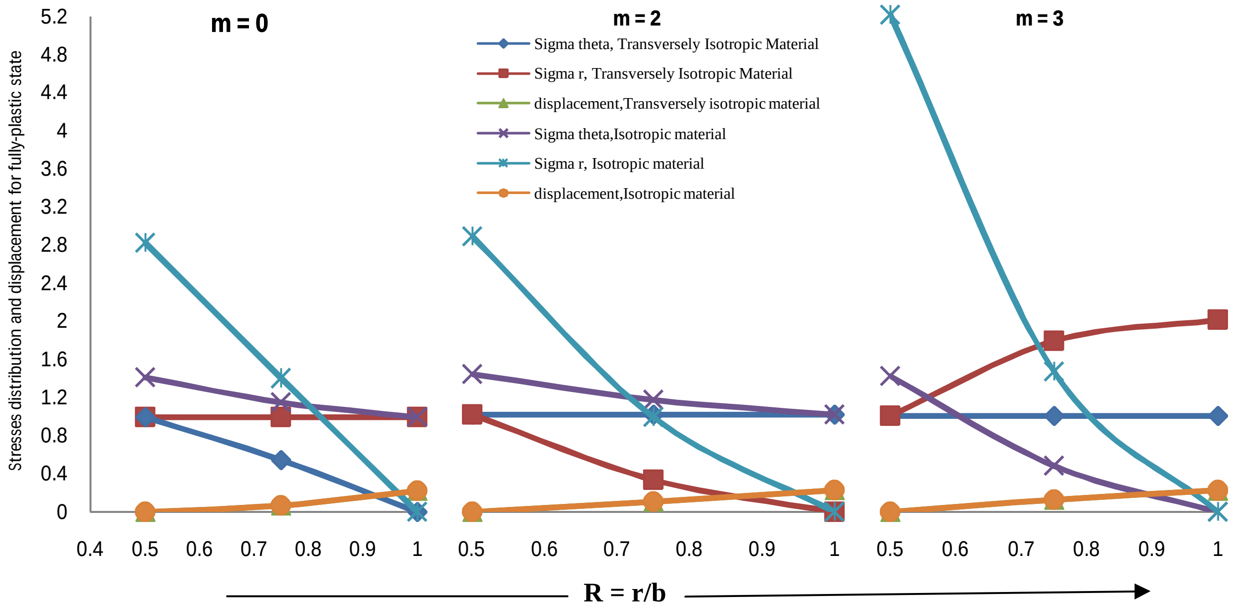


**Figure 4** Stresses distribution and displacement for fully-plastic state along the radius ratio R = r/b.


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